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NATFREQ – A COMPUTER PROGRAM FOR CALCULATING THE NATURAL FREQUENCY OF ROTATING CANTILEVERED BEAMS

by

Joseph B. Wilkerson

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A computer program was developed to evaluate the natural frequencies of model helicopter rotor blades for use in wind tunnel evaluations. This program, NATFREQ, calculates the uncoupled natural frequency and corresponding mode shape of the first, second, and third natural bending modes for a nonuniform cantilever beam rotating in a vacuum. The program includes centrifugal stiffening effects and allows for arbitrary radial

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distributions of blade mass and stiffness properties. The fundamental uncoupled torsional natural frequency may also be calculated for a nonrotating cantilevered nonuniform beam. The blade is represented by a series of concentrated masses connected by massless flexures. The method of calculation for both bending and torsion is by the use of influence coefficients and matrix algebra.

Comparison of calculated results for a uniform beam with the exact solution shows that good results are obtained in the fundamental mode for as few as five mass elements. However, calculated frequencies for the second and third modes of vibration show progressive error. This suggests that up to 20 elements should be used for satisfactory results in the higher modes for nonuniform properties.

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ABSTRACT

A computer program was developed to evaluate the natural frequencies of model helicopter rotor blades for use in wind tunnel evaluations. This program, NATFREQ, calculates the uncoupled natural frequency and corresponding mode shape of the first, second, and third natural bending modes for a nonuniform cantilever beam rotating in a vacuum. The program includes centrifugal stiffening effects and allows for arbitrary radial distributions of blade mass and stiffness properties. The fundamental uncoupled torsional natural frequency may also be calculated for a nonrotating cantilevered nonuniform beam. The blade is represented by a series of concentrated masses connected by massless flexures. The method of calculation for both bending and torsion is by the use of influence coefficients and matrix algebra.

Comparison of calculated results for a uniform beam with the exact solution shows that good results are obtained in the fundamental mode for as few as five mass elements. However, calculated frequencies for the second and third modes of vibration show progressive error. This suggests that up to 20 elements should be used for satisfactory results in the higher modes for nonuniform properties.

ADMINISTRATIVE INFORMATION

The work presented herein was authorized and funded by the Naval Air Systems Command (AIR-320) under Program 62211N, Task F32.421.210 and was accomplished in 1970. The David W. Taylor Naval Ship Research and Development Center (DTNSRDC) Work Unit was 1-1619-100. Preparation of this report was funded under Work Unit 1-1619-200.

All data required for the computer program described herein are in U.S. customary units. Measurements of model geometry and frequency measurements were also taken directly in U.S. customary units. Hence, U.S. customary units are the primary units in this report. Metric units are not given in order to avoid confusion with the required program inputs.

INTRODUCTION

The design of helicopter rotor blades requires the ability to analytically predict their natural frequencies. It is also necessary to know the frequencies of model rotor blades, even if they are not dynamically scaled, to avoid resonant amplification conditions. Resonant amplification occurs when the blade natural frequency coincides with an integer multiple of the rotor rotational frequency. The condition is due to the existence of aerodynamic forcing functions which occur at integer multiples of rotational frequency. Operation at such a condition results in excessive blade deflections taking the mode shape corresponding to the frequency of excitation. For magnitudes of forcing functions which are low relative to blade stiffness, this results only in higher stress levels; but for higher magnitudes of forcing functions (occurring at or near the blades fundamental frequency), the resonant amplification dominates the blade motion causing severe fatique stresses as well as affecting aerodynamic performance data.

COMPUTER PROGRAM

GENERAL DESCRIPTION

The computer program NATFREQ was developed by the author to satisfy the above requirements. This program calculates the uncoupled natural frequency and corresponding mode shape of the first, second, and third natural bending modes for a nonuniform cantilever beam rotating in a vacuum. The program includes centrifugal stiffening effects created by spinning of the rotor blade and allows for arbitrary radial distributions of blade mass and stiffness properties. Since this analysis is intended for rotor blades, henceforth the beam length will be referred to as radius and intermediate lengths along the beam will be referred to as radial positions. In helicopter rotor terminology the bending of the beam is taken to be in the out-of-plane direction. That is, the blade bending takes place in a plane perpendicular to the plane of rotation of the rotor as shown in Figure 1. Conditions of precone and aerodynamic dynamic damping are not accounted for.

The first uncoupled torsional natural frequency and mode shape is also calculated for the cantilevered nonuniform beam. Rotor rotational effects on the blade torsional frequency have not been included in the analysis since they generally have small contributions at the relatively high torsional frequencies of model rotor blades.

ANALYSIS

The method of calculation for both bending and torsion is by the use of influence coefficients and matrix algebra as outlined in Hurty and Rubinstein.[1] The blade is considered to consist of a series of concentrated masses connected by massless flexures; see Figure 2. Root end conditions of the blade are taken at the radius r_o, which is adjusted to suit the particular geometry to be modeled. Blade mass locations r_1, r_2, ..., r_n from the center of rotation are arbitrary and do not have to be evenly spaced. Flexures between the masses may have any distribution of stiffness, but an individual flexure is considered to have uniform stiffness over its length for evaluation of the influence coefficients. The accuracy of results therefore depends on the number of masses and flexures used in the model, as more mass elements provide better representation of even a uniform beam.

Representation of the blade for torsional calculations is similar to that for bending. Concentrated mass polar moments of inertia are connected by torsional springs, as shown in Figure 3. The radius r_o for the root end condition and the radial positions of the polar moments of inertia are taken to be the same numerical values as used for the blade bending calculations. As previously mentioned, the torsion mode is uncoupled from the bending mode and is calculated only for nonrotating conditions.

CENTRIFUGAL STIFFENING

The equations of motion for bending of the nonrotating blade may be expressed in matrix form as

---

[1] Hurty, W.C. and M.F. Rubinstein, "Dynamics of Structures," Prentice-Hall, Inc., Englewood Cliffs, N.J. (1964) pp. 110-136.

$$[m]\{\ddot{\delta}\} + [K]\{\delta\} = \{0\}$$

where [K] = the square symmetric stiffness matrix

[m] = the diagonal mass matrix

$\{\delta\}$ = the column matrix of deflections

At each mass element centrifugal forces act to restore the blade to its undeflected position; see Figure 4. The effect of the restoring force F_i is compounded by the moment created about position r_{i-1} and other inboard elements. This restoring force, proportional to the square of rotational speed, has a considerable impact on the blade out-of-plane bending frequencies. The centrifugal force acting at mass element m_i is

$$CF_i = m_i\, r_i\, \Omega^2$$

This force exerts a restoring force component perpendicular to the blade, shown in Figure 4, which approximately

$$F_i = CF_i \sin \Theta_i$$

where $\sin \Theta_i = \delta_i/\sqrt{r_i^2 + \delta_i^2}$. For small deflections, $\delta_i << r_i$, this reduces to

$$\sin \Theta_i = \delta_i/r_i$$

which gives the restoring force to be

$$F_i = m_i\, \Omega^2\, \delta_i$$

This restoring force is added to the matrix equation of motion to give

$$[m]\{\ddot{\delta}\} + [K]\{\delta\} + \Omega^2 [m]\{\delta\} = \{0\}$$

for the governing equation of natural vibration for the rotating blade.

Solution of the equation of motion is accomplished by matrix algebra to give the characteristic eigenvalue (frequency) and eigenvector (mode shape). The method naturally gives the lowest, or fundamental, frequency which will satisfy the equation for natural harmonic motion. The second mode shape and frequency is a forced solution, requiring it to be orthogonal to the first mode. The third mode shape and frequency is then forced, requiring it to be orthogonal to both the first and second

modes. This is accomplished by the sweeping matrices of reference 1. Since the higher modes are based on the fundamental, they reflect accumulated error and are therefore less accurate than the fundamental.

INPUT/OUTPUT

There are two options within the NATFREQ program. The first option is the most accurate, allowing specific distributions of mass and stiffness properties to be read in. This requires that these distributions be precalculated from detailed engineering drawings or available from some other source. The second option is used for preliminary estimation (or for quick approximations) of frequencies and mode shapes when less information is available. This option requires only a minimum of inputs and uses programmed equations to internally estimate distributions of mass and stiffness. The procedure for calculating frequency is the same for either option once the property distributions are obtained.

Input format for the first option is shown in Table 1. The labels shown in Figure 5 will help to identify and understand the inputs of Table 1.

Input format for the second option is shown in Table 2. Use of this option requires only blade root (RO) and tip (R) general geometry. Mass and stiffness properties are calculated by internal function statements which are defined at the beginning of the program. Those in the current version calculate radial distributions for an elliptical blade cross section with the ellipse major axis in the plane of rotation. Thickness ratio and chord length are considered to be linear with radius from root (RO) to tip (R). The type of blade sections for which this program was written had a hollow cross-sectional shape, shown in Figure 6. First approximations to natural frequency were obtained for this shape by assuming solid elliptical properties and then subtracting those properties attributed to the hollow, or cut out, cross-sectional area. Several inputs of this option are devoted to this purpose and are described in Table 2.

Use of the second option for beams of solid elliptical cross section is easily done by setting all of the cut out inputs to zero. Use of this option for beams with cross-sectional geometry other than elliptical will

require changing the program function statements to equations which are appropriate for the geometry.

Groups of data may be submitted back to back with either program option. The program will read the first data set, execute, return to read a second data set, and so on. Normal exit is produced by putting a card at the end of the last data set with a zero in column 5. (The program exits when the variable N is read in as zero.) A complete program listing of NATFREQ is included in the appendix.

A typical output for each of the two options is shown in Tables 3 and 4. Table 3 (for the first option) shows a printout of the input distributions and the influence coefficient matrices for bending deflection, bending slope, and torsional deflection. The natural frequency and mode shape follow for each of the first three bending modes and for the first torsion mode, in that order. The output frequency has the units of radians per second, and the mode shape is normalized to 1.0 at the last mass point XD(N) rather than at tip R.

Table 4 (for the second option) shows the printout of input blade geometry and material properties, followed by the calculated distributions of blade mass and stiffness properties. The rest of the output is the same as for the first option described above.

RESULTS

BENDING

A lumped mass model for calculation of natural frequency allows for better description and greater accuracy of results for the general case of nonuniform mass and stiffness distribution. However, some judgment is required in modeling the beam to be analyzed to ensure that a reasonable representation is provided to the program. A nonrotating beam with uniform mass and stiffness distribution was analyzed as a check case to evaluate program accuracy and the sensitivity of results to the number of mass elements used. The results (shown in Figure 7) indicate that 20 mass stations are required to give near exact results for the fundamental frequency of the uniform beam; but even 5 elements can give very close

results. For this same number of stations the second and third natural frequencies show progressive error, indicating that 10 stations or more should be used for good accuracy in the higher modes. Figure 8 shows the calculated mode shape for 10 mass elements and for 20 mass elements compared to the exact mode shape for a uniform beam. As shown, the calculated mode shapes are in excellent agreement even for 10 mass elements for the first two modes. Note that these modes have been adjusted to be normalized to 1.0 at blade tip R for comparison to the exact solution.

The above adjustment is easily done by assuming that the mode shape is linear near the blade tip (a good approximation for the first three mode shapes of cantilevered beams). The calculated mode shape is adjusted by linear extrapolation of the two end points to estimate the deflection at tip R by

$$\delta_R = \delta_n + (\delta_n - \delta_{n-1}) \frac{(R-r_n)}{(r_n - r_{n-1})}$$

where δ_n is the last value from the output mode shape (1.0) and δ_{n-1} is the next to last value. The new mode shape, δ'_i, is then obtained from

$$\delta'_i = \delta_i / \delta_R$$

The program has been applied to the calculation of several model rotor blades. One such application was for the blade with the root and tip cross-sectional geometry shown in Figure 6. These blades incorporated 8.6 degrees of geometric twist, as depicted in the figure, and were tapered in thickness ratio. The cantilevered root end began at $r_o = 0.333$ feet and the blade tip was at $R = 3.333$ feet. Mass and stiffness properties were calculated from the detailed engineering drawings for 20 stations. The results from NATFREQ for these blades are shown in Table 5 as compared to the blades experimentally measured nonrotating frequencies. Two of these blades were manufactured; the measured frequencies from both blades are shown. Table 5 shows excellent to good agreement between the measured and calculated natural frequencies.

TORSION

Torsional natural frequency was also checked against the measured frequency of the above-mentioned blades. As shown in Figure 6 the model blades had a hollow duct along the entire blade length with an open slot at the rear edge of the duct. This slot made the blades behave similarly to an open, or C, section. Partial closure was provided by discrete posts (near the duct rear edge) which were spaced out along the blade radius. Therefore, effective values of J estimated along the blade span were used in NATFREQ for torsional frequency calculations. The comparison of calculated to measured torsional frequency is shown in Table 5. Although the comparison is not as good as for blade bending, it is acceptable considering the effective J values which were used.

Torsional calculations were also made for a uniform beam as a check case. Figure 9 shows these results as they vary with the number of concentrated inertias used to represent the beam. The fundamental mode shows only 0.4-percent error from the exact uniform beam solution for a 5 inertia element representation. This suggests that only 5 to 10 inertia stations may be required for good accuracy when calculating the natural frequency of most nonuniform beams.

CONCLUDING REMARKS

The computer program NATFREQ has been shown to be quite accurate for the cases of uncoupled bending and torsional natural frequencies of uniform beams. Calculated natural frequencies for a model rotor blade also showed good agreement with measured values. Care should be exercised, however, to ensure that the program is provided realistic physical inputs when modeling any beam. For instance, beams which have sudden changes in stiffness or mass in some region should have more mass (flexure) elements in the model for that region. Also, beams which have a point mass at some radius in addition to their distributed mass should be modeled accordingly. An additional mass element should be used at the appropriate radius in lieu of distributing the point mass into the adjacent beam elements.

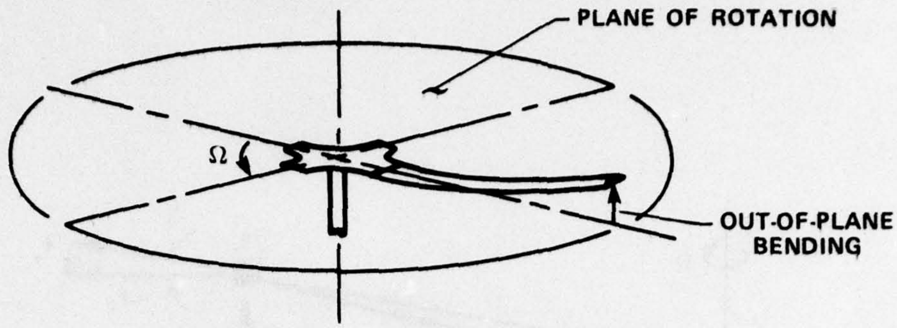

Figure 1 – Blade Bending Diagram

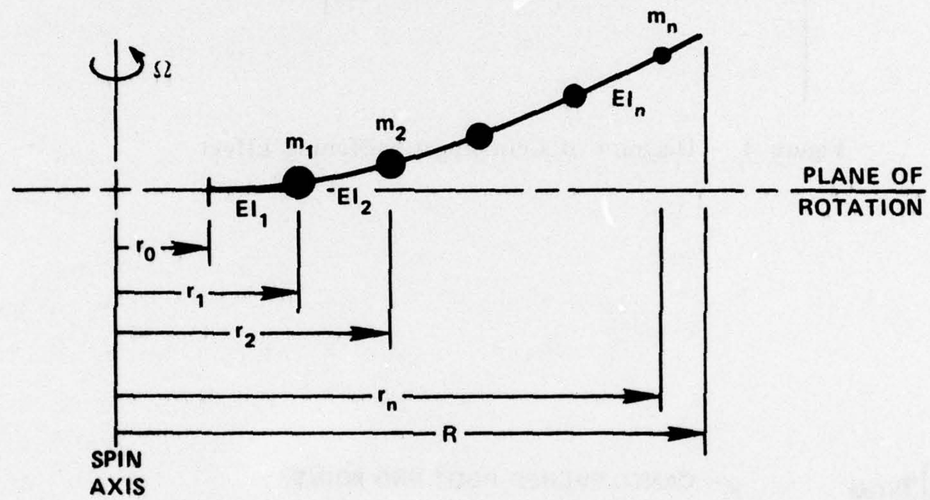

Figure 2 – Discrete Mass Representation

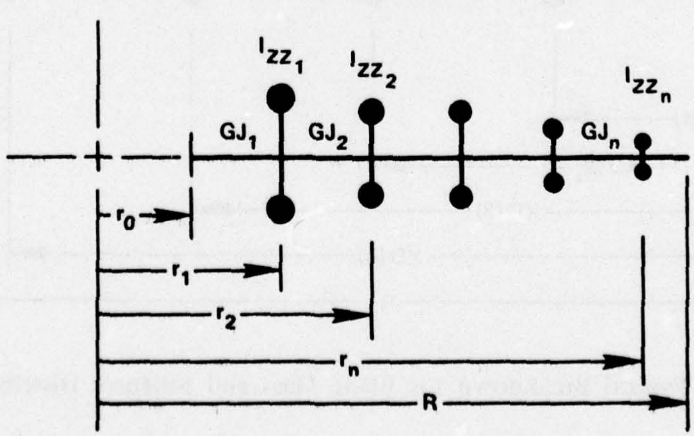

Figure 3 – Blade Torsional Representation

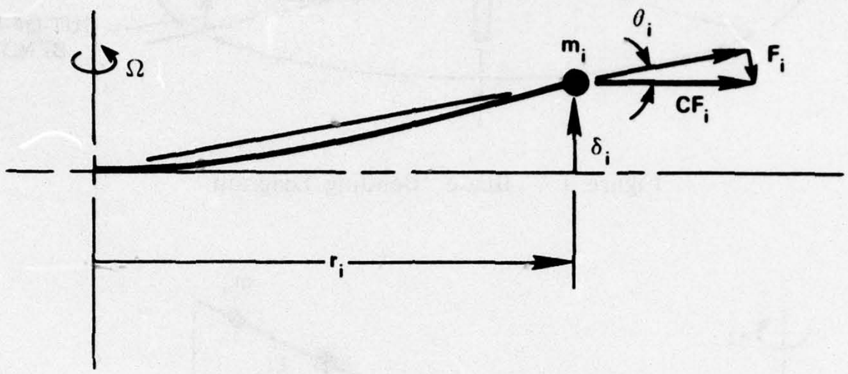

Figure 4 – Diagram of Centrifugal Stiffening Effect

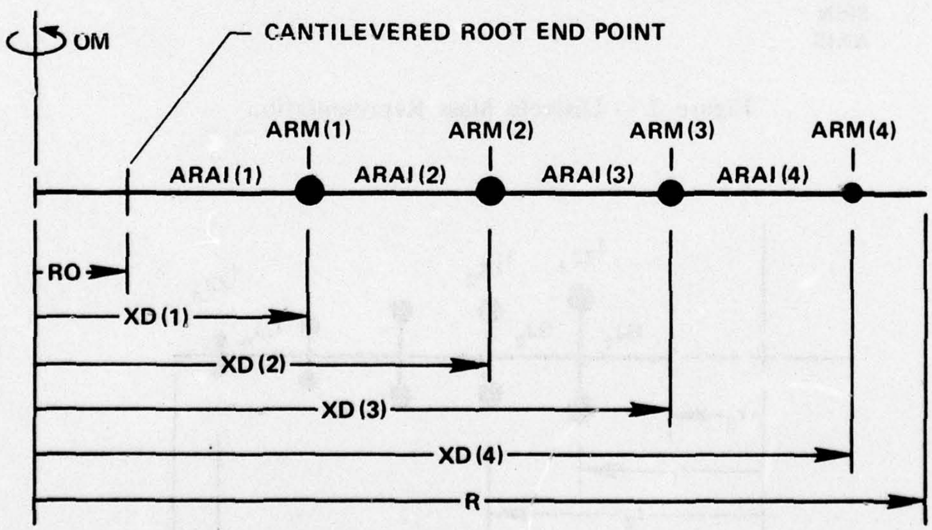

Figure 5 – Typical Breakdown for Blade Mass and Stiffness Distributions

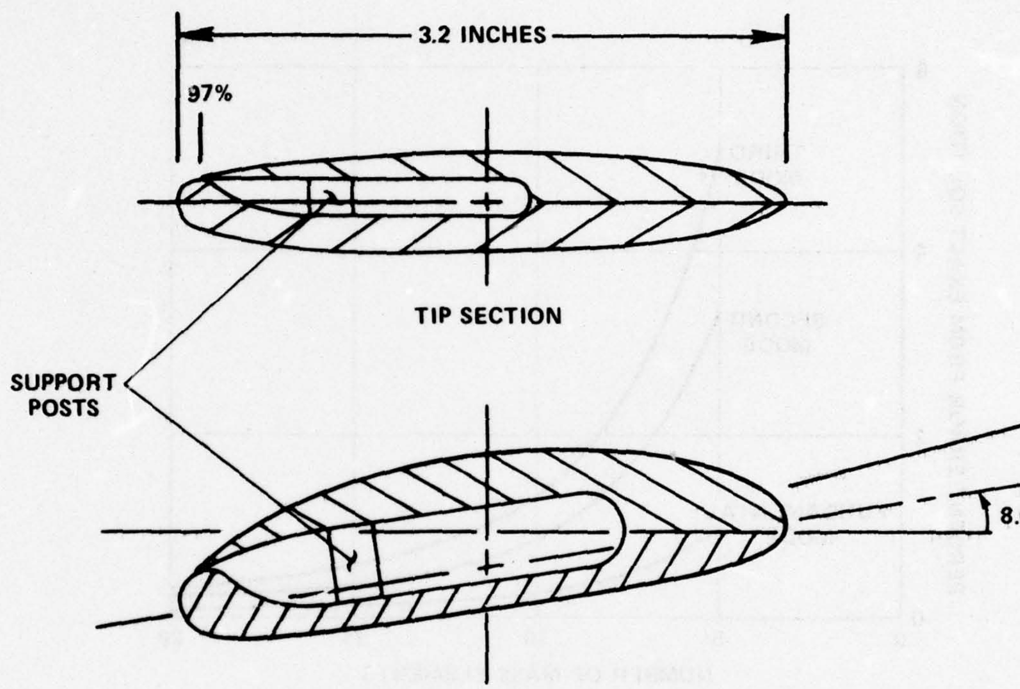

Figure 6 – Model Blade Cross Sections

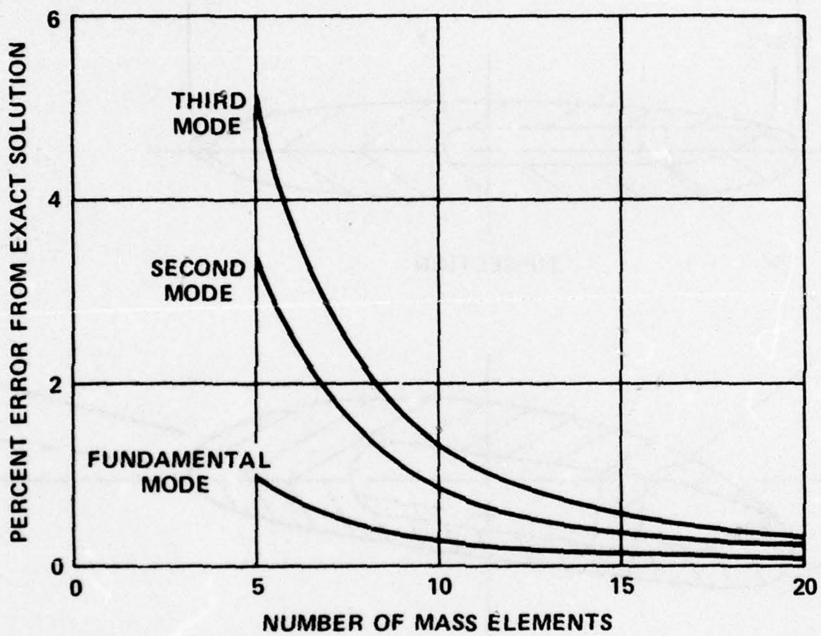

Figure 7 – Comparison of Calculated Natural Frequency with Exact Solution for a Uniform Beam

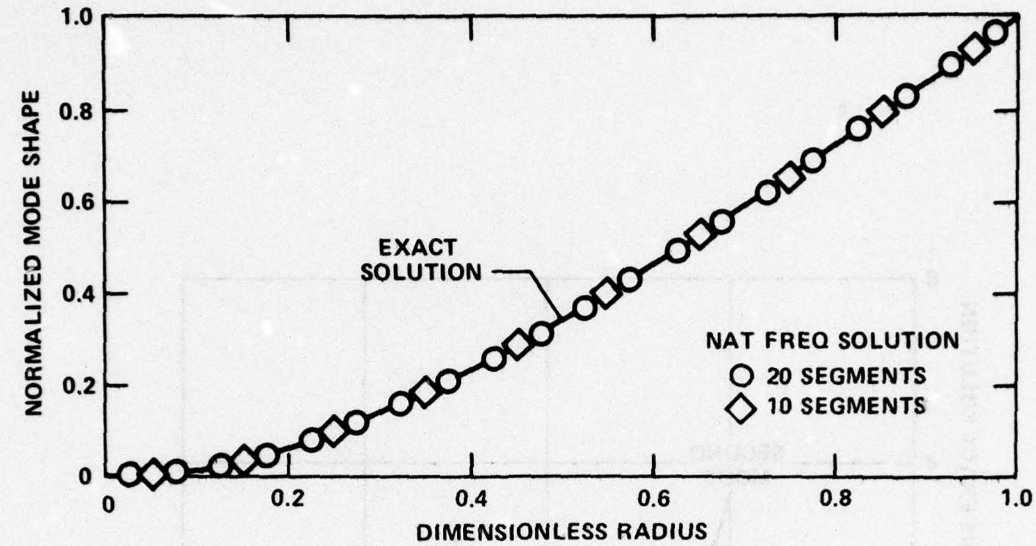

Figure 8a – Fundamental Mode

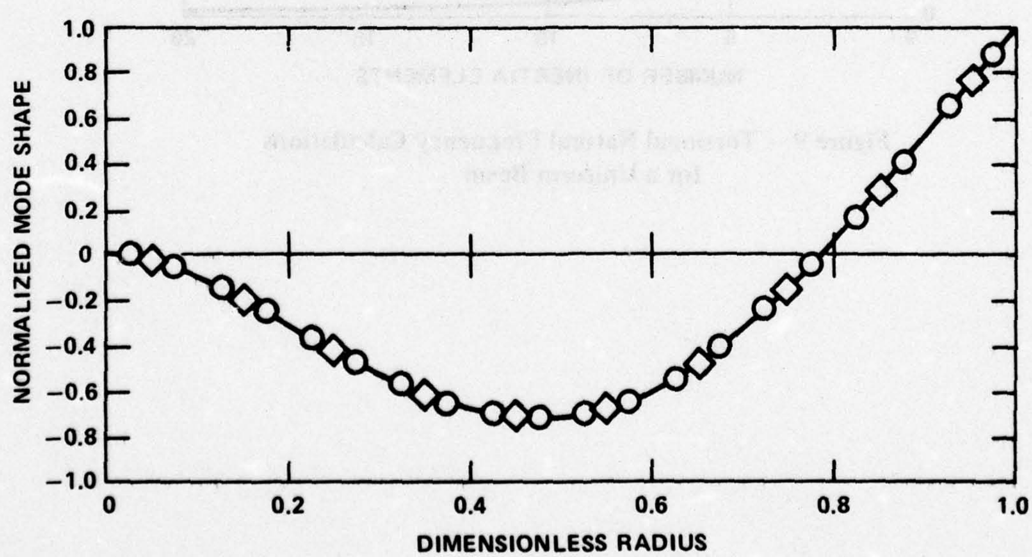

Figure 8b – Second Mode

Figure 8 – Comparison of Calculated Mode Shapes with Exact Solution for a Uniform Beam

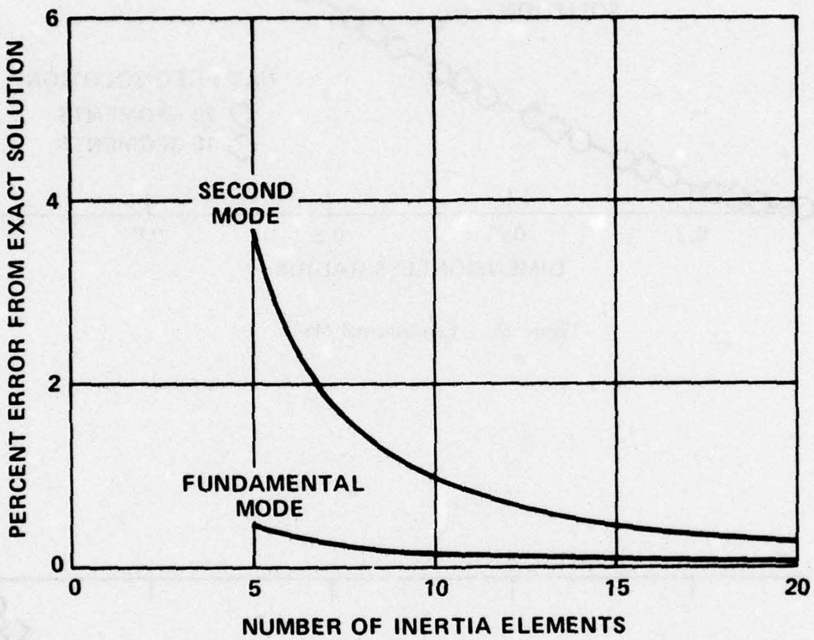

Figure 9 – Torsional Natural Frequency Calculations for a Uniform Beam

TABLE 1 - NATFREQ INPUTS FOR FIRST OPTION

Format	Variable	Description
I5 (one card)	N	Number of mass stations (not to exceed 21)
I5 (one card)	IO	Option control (1 for this table)
5E12.5 (up to 5 cards)	ARAI	Array of N section area moments of inertia, ft^4
5E12.5 (up to 5 cards)	ARAJ*	Array of N section area polar moments of inertia, ft^4
5E12.5 (up to 5 cards)	ARM	Array of N discrete mass weights, lb
5E12.5 (up to 5 cards)	ARP*	Array of N discrete mass polar moments of inertia, lb/ft^2
5E12.5 (up to 5 cards)	XD	Array of N mass stations from center of rotation, ft
3E10.4, 3F10.2 (one card)	E	Youngs modulas of material, lb/ft^2
	G**	Shear modulas of material, lb/ft^2
	DEN	Density of material, lb/ft^3
	OM	Rotor rotational frequency, rad/sec
	R	Blade tip radius from center of rotation, ft
	RO	Blade root radius from center of rotation to root end condition, ft

*These values may be set equal to 1.0 if torsional calculations are not desired; they should not be set to zero.

**Set G to a large number (i.e., 10^{15}) if torsional calculations are not desired.

TABLE 2 - NATFREQ INPUTS FOR SECOND OPTION

Format	Variable	Description
I5 (one card)	N	Number of mass stations (not to exceed 21)
I5 (one card)	IO	Option control (0 for this table)
4F10.2 (one card)	TCR	Blade root thickness ratio (taken at RO)
	TCT	Blade tip thickness ratio (taken at R)
	CR	Blade root chord (in-plane dimension), ft
	CT	Blade tip chord (in-plane dimension), ft
4F10.2 (one card)	R	Blade tip radius from center of rotation, ft
	RO	Blade root radius from center of rotation to root end condition, ft
	ATP	Area of cross-sectional cut out at blade tip, ft^2
	ART	Area of cross-sectional cut out at blade root radius, ft^2
5E12.5 (one card)	ZMIT	Area moment of inertia of cut out area at blade tip, ft^4
	ZMIR	Area moment of inertia of cut out area at blade root radius, ft^4
	ZMJT	Polar area moment of inertia of cut out area at blade tip, ft^4
	ZMJR	Polar area moment of inertia of cut out area at blade root radius, ft^4
	FJ	Empirical correction factor to obtain torsional J term from the calculated section polar area moment of inertia
3E10.4, F10.2 (one card)	E	Youngs modulas of material, lb/ft^2
	G	Shear modulas of material, lb/ft^2
	DEN	Density of material, lb/ft^3
	OM	Rotor rotational frequency, rad/sec

TABLE 3 - NATFREQ OUTPUT FOR FIRST OPTION

MATERIAL PROPERTIES

DENSITY = 1.000000 E = 2.00000E 06 G = 5.00000E 08

FLIGHT CONDITIONS

OMEGA = .CO

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TABLE 3 (Continued)

BLADE CHARACTERISTICS AT COLLOCATION POINTS

X	IXX(X)	WT(X)	J(X)	IZZ(X),POLAR
.408	.9629E-05	.5273E 00	.4694E-05	.8388E-03
.558	.9365E-05	.5221E 00	.4669E-05	.8329E-03
.708	.9023E-05	.5169E 00	.4636E-05	.8270E-03
.858	.8692E-05	.5118E 00	.4603E-05	.8211E-03
1.008	.8372E-05	.5066E 00	.4570E-05	.8153E-03
1.158	.8062E-05	.5014E 00	.4538E-05	.8096E-03
1.308	.7762E-05	.4962E 00	.4506E-05	.8039E-03
1.458	.7473E-05	.4910E 00	.4475E-05	.7983E-03
1.608	.7193E-05	.4858E 00	.4444E-05	.7928E-03
1.758	.6924E-05	.4806E 00	.4413E-05	.7872E-03
1.908	.6665E-05	.4754E 00	.4382E-05	.7818E-03
2.058	.6415E-05	.4702E 00	.4352E-05	.7764E-03
2.208	.6175E-05	.4650E 00	.4322E-05	.7710E-03
2.358	.5944E-05	.4598E 00	.4292E-05	.7657E-03
2.508	.5723E-05	.4546E 00	.4262E-05	.7605E-03
2.658	.5511E-05	.4494E 00	.4233E-05	.7552E-03
2.808	.5307E-05	.4442E 00	.4204E-05	.7501E-03
2.958	.5113E-05	.4390E 00	.4175E-05	.7450E-03
3.108	.4928E-05	.4338E 00	.4147E-05	.7399E-03
3.258	.4751E-05	.4286E 00	.41:9E-05	.7349E-03

BLADE WEIGHT = 9.56

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TABLE 3 (Continued)

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INFLUENCE COEFFICIENT MATRICES

DEFLECTION - AIC(I,J)

.15734E-05	.24405E-05	.33075E-05	.41746E-05	.50417E-05	.59087E-05	.67758E-05	.76428E-05	.85099E-05	.93770E-05
.24405E-05	.40248E-05	.56493E-05	.72737E-05	.88982E-05	.10523E-04	.12147E-04	.13772E-04	.15396E-04	.17020E-04
.33075E-05	.56493E-05	.82347E-05	.10862E-04	.13489E-04	.16116E-04	.18743E-04	.21370E-04	.23997E-04	.26624E-04
.41746E-05	.72737E-05	.10862E-04	.14703E-04	.18587E-04	.22471E-04	.26355E-04	.30239E-04	.34123E-04	.38007E-04
.50417E-05	.88982E-05	.13489E-04	.18587E-04	.23947E-04	.29353E-04	.34758E-04	.40164E-04	.45569E-04	.50975E-04
.59087E-05	.10523E-04	.16116E-04	.22471E-04	.29353E-04	.36507E-04	.43708E-04	.50910E-04	.58111E-04	.65312E-04
.67758E-05	.12147E-04	.18743E-04	.26355E-04	.34758E-04	.43708E-04	.52942E-04	.62224E-04	.71505E-04	.80787E-04
.76428E-05	.13772E-04	.21370E-04	.30239E-04	.40164E-04	.50910E-04	.62224E-04	.73832E-04	.85490E-04	.97149E-04
.85099E-05	.15396E-04	.23997E-04	.34123E-04	.45569E-04	.58111E-04	.71505E-04	.85490E-04	.99780E-04	.11412E-03
.93770E-05	.17020E-04	.26624E-04	.38007E-04	.50975E-04	.65312E-04	.80787E-04	.97149E-04	.11412E-03	.13141E-03
.10244E-04	.18645E-04	.29251E-04	.41891E-04	.56380E-04	.72513E-04	.90069E-04	.10881E-03	.12847E-03	.14876E-03
.11111E-04	.20269E-04	.31878E-04	.45776E-04	.61785E-04	.79714E-04	.99351E-04	.12047E-03	.14281E-03	.16611E-03
.11978E-04	.21894E-04	.34505E-04	.49660E-04	.67191E-04	.86915E-04	.10863E-03	.13212E-03	.15715E-03	.18345E-03
.12845E-04	.23518E-04	.37132E-04	.53544E-04	.72596E-04	.94117E-04	.11791E-03	.14378E-03	.17149E-03	.20080E-03
.13712E-04	.25143E-04	.39759E-04	.57428E-04	.78002E-04	.10132E-03	.12720E-03	.15544E-03	.18584E-03	.21814E-03
.14579E-04	.26767E-04	.42386E-04	.61312E-04	.83407E-04	.10852E-03	.13648E-03	.16710E-03	.20018E-03	.23549E-03
.15446E-04	.28392E-04	.45013E-04	.65196E-04	.88813E-04	.11572E-03	.14576E-03	.17876E-03	.21452E-03	.25284E-03
.16313E-04	.30016E-04	.47640E-04	.69080E-04	.94218E-04	.12292E-03	.15504E-03	.19042E-03	.22886E-03	.27018E-03
.17180E-04	.31640E-04	.50267E-04	.72964E-04	.99624E-04	.13012E-03	.16432E-03	.20207E-03	.24321E-03	.28753E-03
.18048E-04	.33265E-04	.52894E-04	.76849E-04	.10503E-03	.13732E-03	.17361E-03	.21373E-03	.25755E-03	.30487E-03

- - - - - - CONTINUING ACROSS, INCREASING J - - - - - -

.10244E-04	.11111E-04	.11978E-04	.12845E-04	.13712E-04	.14579E-04	.15446E-04	.16313E-04	.17180E-04	.18048E-04
.18645E-04	.20269E-04	.21894E-04	.23518E-04	.25143E-04	.26767E-04	.28392E-04	.30016E-04	.31640E-04	.33265E-04
.29251E-04	.31878E-04	.34505E-04	.37132E-04	.39759E-04	.42386E-04	.45013E-04	.47640E-04	.50267E-04	.52894E-04
.41891E-04	.45776E-04	.49660E-04	.53544E-04	.57428E-04	.61312E-04	.65196E-04	.69080E-04	.72964E-04	.76849E-04
.56380E-04	.61785E-04	.67191E-04	.72596E-04	.78002E-04	.83407E-04	.88813E-04	.94218E-04	.99624E-04	.10503E-03
.72513E-04	.79714E-04	.86915E-04	.94117E-04	.10132E-03	.10852E-03	.11572E-03	.12292E-03	.13012E-03	.13732E-03
.90069E-04	.99351E-04	.10863E-03	.11791E-03	.12720E-03	.13648E-03	.14576E-03	.15504E-03	.16432E-03	.17361E-03
.10881E-03	.12047E-03	.13212E-03	.14378E-03	.15544E-03	.16710E-03	.17876E-03	.19042E-03	.20207E-03	.21373E-03
.12847E-03	.14281E-03	.15715E-03	.17149E-03	.18584E-03	.20018E-03	.21452E-03	.22886E-03	.24321E-03	.25755E-03
.14876E-03	.16611E-03	.18345E-03	.20080E-03	.21814E-03	.23549E-03	.25284E-03	.27018E-03	.28753E-03	.30487E-03
.16939E-03	.19007E-03	.21075E-03	.23143E-03	.25211E-03	.27279E-03	.29347E-03	.31415E-03	.33483E-03	.35552E-03
.19007E-03	.21437E-03	.23873E-03	.26309E-03	.28745E-03	.31181E-03	.33617E-03	.36054E-03	.38490E-03	.40926E-03
.21075E-03	.23873E-03	.26707E-03	.29547E-03	.32387E-03	.35227E-03	.38067E-03	.40906E-03	.43746E-03	.46586E-03
.23143E-03	.26309E-03	.29547E-03	.32821E-03	.36102E-03	.39383E-03	.42664E-03	.45945E-03	.49226E-03	.52507E-03
.25211E-03	.28745E-03	.32387E-03	.36102E-03	.39856E-03	.43617E-03	.47378E-03	.51138E-03	.54899E-03	.58659E-03
.27279E-03	.31181E-03	.35227E-03	.39383E-03	.43617E-03	.47891E-03	.52171E-03	.56451E-03	.60732E-03	.65012E-03
.29347E-03	.33617E-03	.38067E-03	.42664E-03	.47378E-03	.52171E-03	.57006E-03	.61848E-03	.66689E-03	.71531E-03
.31415E-03	.36054E-03	.40906E-03	.45945E-03	.51138E-03	.56451E-03	.61848E-03	.67287E-03	.72733E-03	.78180E-03
.33483E-03	.38490E-03	.43746E-03	.49226E-03	.54899E-03	.60732E-03	.66689E-03	.72733E-03	.78822E-03	.84918E-03
.35552E-03	.40926E-03	.46586E-03	.52507E-03	.58659E-03	.65012E-03	.71531E-03	.78180E-03	.84918E-03	.91703E-03

TABLE 3 (Continued)

SLOPE • BIC[I,J]

.578C4E-05	.10028E-04	.14275E-04	.18522E-04	.22769E-04	.27016E-04	.31263E-04	.35511E-04	.39758E-04	.44005E-04
.578C4E-C5	.10830E-04	.16681E-04	.22532E-04	.28384E-04	.34235E-04	.40086E-04	.45938E-04	.51789E-04	.57641E-04
.57804E-05	.10830E-04	.17514E-04	.25030E-04	.32546E-04	.40063E-04	.47579E-04	.55095E-04	.62612E-04	.70128E-04
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.35139E-04	.44384E-04	.53629E-04	.62874E-04	.72118E-04	.81363E-04
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.47076E-04	.58115E-04	.69155E-04	.80194E-04	.91234E-04
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.60911E-04	.73814E-04	.86717E-04	.99620E-04
.578C4E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.76717E-04	.91556E-04	.10640E-03
.57804E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.94572E-04	.11142E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11455E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.57804E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.578C4E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.57804E-05	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03
.57804E-C5	.10830E-04	.17514E-04	.25894E-04	.36036E-04	.48008E-04	.61879E-04	.77723E-04	.95616E-04	.11564E-03

• • • • • • CONTINUING ACROSS, INCREASING J • • • • • •

.48252E-C4	.52499E-04	.56746E-04	.60994E-04	.65241E-04	.69488E-04	.73735E-04	.77982E-04	.82229E-04	.86477E-04
.63492E-C4	.69343E-04	.75195E-04	.81046E-04	.86897E-04	.92749E-04	.98600E-04	.10445E-03	.11030E-03	.11615E-03
.77645E-C4	.85161E-04	.92677E-04	.10019E-03	.10771E-03	.11523E-03	.12274E-03	.13026E-03	.13778E-03	.14529E-03
.9C6C8E-04	.99853E-04	.10910E-03	.11834E-03	.12759E-03	.13683E-03	.14608E-03	.15532E-03	.16457E-03	.17381E-03
.1C227E-03	.11331E-03	.12435E-03	.13539E-03	.14643E-03	.15747E-03	.16851E-03	.17955E-03	.19059E-03	.20163E-03
.11252E-03	.12543E-03	.13833E-03	.15123E-03	.16414E-03	.17704E-03	.18994E-03	.20285E-03	.21575E-03	.22865E-03
.12123E-03	.13607E-03	.15091E-03	.16575E-03	.18059E-03	.19543E-03	.21027E-03	.22511E-03	.23994E-03	.25478E-03
.12827E-03	.14512E-03	.16197E-03	.17882E-03	.19567E-03	.21252E-03	.22937E-03	.24622E-03	.26307E-03	.27992E-03
.13349E-03	.15243E-03	.17137E-03	.19031E-03	.20924E-03	.22818E-03	.24712E-03	.26606E-03	.28500E-03	.30393E-03
.13675E-03	.15785E-03	.17896E-03	.20007E-03	.22118E-03	.24229E-03	.26339E-03	.28450E-03	.30561E-03	.32672E-03
.13787E-03	.16124E-03	.18460E-03	.20796E-03	.23132E-03	.25468E-03	.27805E-03	.30141E-03	.32477E-03	.34813E-03
.13787E-03	.16241E-03	.18811E-03	.21382E-03	.23952E-03	.26522E-03	.29093E-03	.31663E-03	.34234E-03	.36804E-03
.13787E-03	.16241E-03	.18933E-03	.21747E-03	.24560E-03	.27374E-03	.30188E-03	.33001E-03	.35815E-03	.38629E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.24939E-03	.28006E-03	.31072E-03	.34139E-03	.37205E-03	.40272E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28400E-03	.31729E-03	.35058E-03	.38387E-03	.41716E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28536E-03	.32138E-03	.35739E-03	.39341E-03	.42942E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28536E-03	.32279E-03	.36164E-03	.40049E-03	.43933E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28536E-03	.32279E-03	.36311E-03	.40489E-03	.44668E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28536E-03	.32279E-03	.36311E-03	.40642E-03	.45125E-03
.13787E-03	.16241E-03	.18933E-03	.21873E-03	.25071E-03	.28536E-03	.32279E-03	.36311E-03	.40642E-03	.45283E-03

BEST AVAILABLE COPY

BEST AVAILABLE COPY

TABLE 3 (Continued)

TORSION - CIC(I,J)

.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04
.28451E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04
.28451E-04	.85660E-04	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.43756E-03	.43756E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.49766E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03
.28451E-04	.85660E-04	.14328E-03	.20130E-03	.25974E-03	.31860E-03	.37787E-03	.43756E-03	.49766E-03	.55819E-03

. . . . . . CONTINUING ACROSS, INCREASING J . . . . . .

.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04	.28451E-04
.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04	.85660E-04
.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03	.14328E-03
.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03	.20130E-03
.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03	.25974E-03
.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03	.31860E-03
.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03	.37787E-03
.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03	.43756E-03
.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03	.49766E-03
.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03	.55819E-03
.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03	.61914E-03
.61914E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03	.68052E-03
.61914E-03	.68052E-03	.74232E-03	.74232E-03	.74232E-03	.74232E-03	.74232E-03	.74232E-03	.74232E-03	.74232E-03
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.80456E-03	.80456E-03	.80456E-03	.80456E-03	.80456E-03	.80456E-03
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.86722E-03	.86722E-03	.86722E-03	.86722E-03	.86722E-03
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.93032E-03	.93032E-03	.93032E-03	.93032E-03	.93032E-03
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.93032E-03	.99385E-03	.99385E-03	.99385E-03	.99385E-03
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.93032E-03	.99385E-03	.10578E-02	.10578E-02	.10578E-02
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.93032E-03	.99385E-03	.10578E-02	.11222E-02	.11222E-02
.61914E-03	.68052E-03	.74232E-03	.80456E-03	.86722E-03	.93032E-03	.99385E-03	.10578E-02	.11222E-02	.11871E-02

TABLE 3 (Continued)

MODE SHAPE 1   NAT. FREQ = .24092E 03   ITERATIONS = 5

.C2305124
.C4222609
.C6670711
.C9625332
.13060271
.16947261
.21256059
.25954582
.31009107
.36384532
.42044712
.47952869
.54072084
.60365883
.66798909
.73337703
.79951580
.86613619
.93301757
1.00000000

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TABLE 3 (Continued)

MODE SHAPE 2 NAT. FREQ = .70022E 03 ITERATIONS = 5

-.12050015
-.20760273
-.30607719
-.40853688
-.50759846
-.59610216
-.66735474
-.71538162
-.73517373
-.72291241
-.67615545
-.59396685
-.47697355
-.32733367
-.14860322
.05450906
.27650529
.51158332
.75424166
1.00000000

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TABLE 3 (Continued)

MODE SHAPE 3   NAT. FREQ = .18305E 04   ITERATIONS = 8

.30029127
.47858574
.63982640
.75502062
.80187451
.76717755
.64848616
.45476148
.2057398C
-.C7003201
-.33838337
-.56362826
-.71312252
-.76166854
-.6951C161
-.51241067
-.22585161
.14130373
.5595C230
1.0000000C

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TABLE 3 (Continued)

TORSION MODE 1 NAT. FREQ = .95343E 03 ITERATIONS = 4

.C3772808
.11334218
.18875493
.26347001
.33699214
.4C883040
.4785C145
.54553284
.60946614
.66986010
.72629366
.77836890
.82571378
.86798483
.90486954
.93608873
.96139850
.98059219
.9935C192
1.C000C000

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TABLE 4 - NATFREQ OUTPUT FOR SECOND OPTION

```
                      BLADE GEOMETRY

ROOT T/C =  .200            TIP T/C =  .150
ROOT CHORD =      .42       TIP CHORD =      .42
ROOT RADIUS =      .33      TIP RADIUS =     3.33


                    MATERIAL PROPERTIES

DENSITY =  172.800000      E =  1.49760E 09       G =  5.61600E 08


                    FLIGHT CONDITIONS

OMEGA =  209.44
```

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TABLE 4 (Continued)

BLADE CHARACTERISTICS AT COLLOCATION POINTS

X	IXX(X)	WT(X)	J(X)	IZZ(X),POLAR
.333	.5000E-03	.2145E 01	.5000E-04	.3000E-03
1.COC	.5000E-03	.2145E 01	.5000E-04	.3000E-03
1.667	.5000E-03	.2145E 01	.5000E-04	.3000E-03
2.333	.5000E-03	.2145E 01	.5000E-04	.3000E-03
3.CCC	.5000E-03	.2145E 01	.5000E-04	.3000E-03
3.667	.5000E-03	.2145E 01	.5000E-04	.3000E-03
4.333	.5000E-03	.2145E 01	.5000E-04	.3000E-03
5.000	.5000E-03	.2145E 01	.5000E-04	.3000E-03
5.667	.5000E-03	.2145E 01	.5000E-04	.3000E-03
6.333	.5000E-03	.2145E 01	.5000E-04	.3000E-03
7.COO	.5000E-03	.2145E 01	.5000E-04	.3000E-03
7.667	.5000E-03	.2145E 01	.5000E-04	.3000E-03
8.333	.5000E-03	.2145E 01	.5000E-04	.3000E-03
9.00C	.5000E-03	.2145E 01	.5000E-04	.3000E-03
9.667	.5000E-03	.2145E 01	.5000E-04	.3000E-03

BLADE WEIGHT = 32.17

TABLE 4 (Continued)

INFLUENCE COEFFICIENT MATRICES

DEFLECTION - AIC(I,J)

.12346E-04	.49383E-04	.86420E-04	.12346E-03	.16049E-03	.19753E-03	.23457E-03	.27160E-03	.30864E-03	.34568E-03
.49383E-04	.33333E-03	.66667E-03	.10000E-02	.13333E-02	.16667E-02	.20000E-02	.23333E-02	.26667E-02	.30000E-02
.86420E-04	.66667E-03	.15432E-02	.24691E-02	.33951E-02	.43210E-02	.52469E-02	.61728E-02	.70988E-02	.80247E-02
.12346E-03	.10000E-02	.24691E-02	.42346E-02	.60494E-02	.78642E-02	.96790E-02	.11494E-01	.13309E-01	.15123E-01
.16049E-03	.13333E-02	.33951E-02	.60494E-02	.90000E-02	.12000E-01	.15000E-01	.18000E-01	.21000E-01	.24000E-01
.19753E-03	.16667E-02	.43210E-02	.78642E-02	.12000E-01	.16432E-01	.20914E-01	.25395E-01	.29877E-01	.34358E-01
.23457E-03	.20000E-02	.52469E-02	.96790E-02	.15000E-01	.20914E-01	.27123E-01	.33383E-01	.39642E-01	.45901E-01
.27160E-03	.23333E-02	.61728E-02	.11494E-01	.18000E-01	.25395E-01	.33383E-01	.41667E-01	.50000E-01	.58333E-01
.30864E-03	.26667E-02	.70988E-02	.13309E-01	.21000E-01	.29877E-01	.39642E-01	.50000E-01	.60654E-01	.71358E-01
.34568E-03	.30000E-02	.80247E-02	.15123E-01	.24000E-01	.34358E-01	.45901E-01	.58333E-01	.71358E-01	.84679E-01
.38272E-03	.33333E-02	.89506E-02	.16938E-01	.27000E-01	.38840E-01	.52160E-01	.66667E-01	.82062E-01	.98049E-01
.41975E-03	.36667E-02	.98765E-02	.18753E-01	.30000E-01	.43321E-01	.58420E-01	.75000E-01	.92765E-01	.11142E 00
.45679E-03	.40000E-02	.10802E-01	.20568E-01	.33000E-01	.47802E-01	.64679E-01	.83333E-01	.10347E 00	.12479E 00
.49383E-03	.43333E-02	.11728E-01	.22383E-01	.36000E-01	.52284E-01	.70938E-01	.91667E-01	.11417E 00	.13816E 00
.53086E-03	.46667E-02	.12654E-01	.24198E-01	.39000E-01	.56765E-01	.77198E-01	.10000E 00	.12488E 00	.15153E 00

- - - - - - - CONTINUING ACROSS, INCREASING J - - - - - - -

.38272E-03	.41975E-03	.45679E-03	.49383E-03	.53086E-03
.33333E-02	.36667E-02	.40000E-02	.43333E-02	.46667E-02
.89506E-02	.98765E-02	.10802E-01	.11728E-01	.12654E-01
.16938E-01	.18753E-01	.20568E-01	.22383E-01	.24198E-01
.27000E-01	.30000E-01	.33000E-01	.36000E-01	.39000E-01
.38840E-01	.43321E-01	.47802E-01	.52284E-01	.56765E-01
.52160E-01	.58420E-01	.64679E-01	.70938E-01	.77198E-01
.66667E-01	.75000E-01	.83333E-01	.91667E-01	.10000E 00
.82062E-01	.92765E-01	.10347E 00	.11417E 00	.12488E 00
.98049E-01	.11142E 00	.12479E 00	.13816E 00	.15153E 00
.11433E 00	.13067E 00	.14700E 00	.16333E 00	.17967E 00
.13067E 00	.15021E 00	.16980E 00	.18940E 00	.20899E 00
.14700E 00	.16980E 00	.19290E 00	.21605E 00	.23920E 00
.16333E 00	.18940E 00	.21605E 00	.24300E 00	.27000E 00
.17967E 00	.20899E 00	.23920E 00	.27000E 00	.30110E 00

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TABLE 4 (Continued)

SLOPE = BIC(I,J)

.55556E-04	.27778E-03	.50000E-03	.72222E-03	.94444E-03	.11667E-02	.13889E-02	.16111E-02	.18333E-02	.20556E-02
.55556E-04	.50000E-03	.11667E-02	.18333E-02	.25000E-02	.31667E-02	.38333E-02	.45000E-02	.51667E-02	.58333E-02
.55556E-04	.50000E-03	.13889E-02	.25000E-02	.36111E-02	.47222E-02	.58333E-02	.69444E-02	.80556E-02	.91667E-02
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.42778E-02	.58333E-02	.73889E-02	.89444E-02	.10500E-01	.12056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.65000E-02	.85000E-02	.10500E-01	.12500E-01	.14500E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.91667E-02	.11611E-01	.14056E-01	.16500E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12278E-01	.15167E-01	.18056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.15833E-01	.19167E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.19833E-01
.55556E-04	.50000E-03	.13889E-C2	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01
.55556E-04	.50000E-03	.13889E-C2	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01
.55556E-04	.50000E-03	.13889E-02	.27222E-02	.45000E-02	.67222E-02	.93889E-02	.12500E-01	.16056E-01	.20056E-01

. . . . CONTINUING ACROSS, INCREASING J . . . . . .

.22778E-02	.25000E-02	.27222E-02	.29444E-02	.31667E-02
.65000E-02	.71667E-02	.78333E-02	.85000E-02	.91667E-02
.10278E-01	.11389E-01	.125C0E-01	.13611E-01	.14722E-01
.13611E-01	.15167E-01	.16722E-01	.18278E-01	.19833E-01
.16500E-01	.18500E-01	.20500E-01	.22500E-01	.24500E-01
.18944E-01	.21389E-01	.23833E-01	.26278E-01	.28722E-01
.2C944E-01	.23833E-01	.26722E-01	.29611E-01	.32500E-01
.22500E-01	.25833E-01	.29167E-01	.32500E-01	.35833E-01
.23611E-01	.27389E-01	.31167E-01	.34944E-01	.38722E-01
.24278E-01	.28500E-01	.32722E-01	.36944E-01	.41167E-01
.24500E-C1	.29167E-01	.33833E-01	.38500E-01	.43167E-01
.24500E-C1	.29389E-01	.34500E-01	.39611E-01	.44722E-01
.24500E-C1	.29389E-01	.34722E-01	.40278E-01	.45833E-01
.24500E-01	.29389E-01	.34722E-01	.40500E-01	.46500E-01
.245C0E-01	.29389E-01	.34722E-01	.40500E-01	.46722E-01

TABLE 4 (Continued)

TORSION - CIC(I,J)

.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04
.13333E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04
.13333E-04	.40000E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.93333E-04	.93333E-04	.93333E-04	.93333E-04	.93333E-04	.93333E-04
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.12000E-03	.12000E-03	.12000E-03	.12000E-03	.12000E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.14667E-03	.14667E-03	.14667E-03	.14667E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.17333E-03	.17333E-03	.17333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.20000E-03	.20000E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.22667E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03
.13333E-04	.40000E-04	.66667E-04	.93333E-04	.12000E-03	.14667E-03	.17333E-03	.20000E-03	.22667E-03	.25333E-03

- - - - - - CONTINUING ACROSS, INCREASING J - - - - - -

.13333E-04	.13333E-04	.13333E-04	.13333E-04	.13333E-04
.40000E-04	.40000E-04	.40000E-04	.40000E-04	.40000E-04
.66667E-04	.66667E-04	.66667E-04	.66667E-04	.66667E-04
.93333E-04	.93333E-04	.93333E-04	.93333E-04	.93333E-04
.12000E-03	.12000E-03	.12000E-03	.12000E-03	.12000E-03
.14667E-03	.14667E-03	.14667E-03	.14667E-03	.14667E-03
.17333E-03	.17333E-03	.17333E-03	.17333E-03	.17333E-03
.20000E-03	.20000E-03	.20000E-03	.20000E-03	.20000E-03
.22667E-03	.22667E-03	.22667E-03	.22667E-03	.22667E-03
.25333E-03	.25333E-03	.25333E-03	.25333E-03	.25333E-03
.28000E-03	.28000E-03	.28000E-03	.28000E-03	.28000E-03
.28000E-03	.30667E-03	.30667E-03	.30667E-03	.30667E-03
.28000E-03	.30667E-03	.33333E-03	.33333E-03	.33333E-03
.28000E-03	.30667E-03	.33333E-03	.36000E-03	.36000E-03
.28000E-03	.30667E-03	.33333E-03	.36000E-03	.38667E-03

TABLE 4 (Continued)

MODE SHAPE 1 NAT. FREQ = .35196E 01 ITERATIONS = 3

.CC201723
.C1759061
.C4729586
.C8963364
.14311669
.20628014
.27769452
.35598119
.43982974
.52801707
.61942792
.71307633
.8C812802
.90392323
1.CCOCCOCC

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TABLE 4 (Continued)

```
MODE SHAPE  2     NAT. FREQ =  .22113E 02     ITERATIONS =  5

-.01376090
-.10995131
-.26704605
-.44937140
-.62387363
-.76182681
-.84047212
-.84431359
-.76585278
-.60561375

-.37138610
-.07668969
 .26146585
 .62540021
1.00000000
```

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TABLE 4 (Continued)

MØDE SHAPE 3 NAT. FREQ = .62055E 02 ITERATIONS = 8

.C4186556
.3C494457
.653C236C
.52432265
1.C0574382
.85209195
.49169454
.C1507267
-.45132577
-.77977768
-.87514203
-.69863701
-.2746C161
.32262193
1.COC0000C

TABLE 4 (Continued)

TORSION MODE 1   NAT. FREQ = .14333E 04   ITERATIONS = 4

.C524C737
.15664796
.25917236
.35885733
.45461C74
.54538349
.63C181C9
.7C8C7443
.77821CC9
.83981959
.89222787
.93486C7C
.96725C92
.98904362
1.COCOCOOC

TABLE 5 - COMPARISON OF CALCULATED AND MEASURED NATURAL FREQUENCIES

Mode of Vibration	NATFREQ Calculated (rad/sec)	Measured (rad/sec) Blade 1	Blade 2
1st Bending	121.9	118.7	121.3
2nd Bending	634.9	603.8	612.0
3rd Bending	1470.0	1521.8	1545.7
1st Torsion	919.9	1097.7	1030.4

APPENDIX

NATFREQ PROGRAM LISTING

```
C     PROGRAM  NATFREQ
C     THIS PROGRAM CALCULATES THE FIRST THREE NATURAL BENDING
C         FREQUENCIES OF A NONUNIFORM BEAM ROTATING IN A VACUUM.
C     PROGRAM BY J. B. WILKERSON,  CODE 1619
C
C     FUNCTION STATEMENTS
C   ELLIPTICAL CROSS SECTION, TAPERED THICKNESS RATIO AND PLANFORM
C   THICKNESS RATIO (X)
      TC(X) = (TCT-TCR)*(X-R0)/RP + TCR
C   CHORD (X)
      C(X) = (CT-CR)*(X-R0)/RP + CR
C   DUCT INTERNAL AREA (X)
      AI(X) = (ATP-ART)*(X-R0)/RP + ART
C   DUCT AREA MOMENT OF INERTIA (X), FLAPPING
      ZMI(X) = (ZMIT-ZMIR)*(X-R0)/RP + ZMIR
C   DUCT POLAR AREA MOMENT OF INERTIA (X), TORSION
      ZMJ(X) = (ZMJT-ZMJR)*(X-R0)/RP + ZMJR
C   BLADE MASS (X)
      AMS(X) = DEN*(3.14159*TC(X)*C(X)**2/4.-AI(X) )
C   BLADE AREA MOMENT OF INERTIA (X), FLAPPING
      AMI(X) = 3.14159*TC(X)**3*C(X)**4/64. - ZMI(X)
C   BLADE POLAR AREA MOMENT OF INERTIA(X), TORSION
      AMJ(X) = 3.14159*TC(X)*C(X)**4*(1.+TC(X)**2)/64. - ZMJ(X)
C   BLADE MASS POLAR INERTIA IS CONSIDERED TO BE AREAPOLAR INERTIA*DENSITY
C     END FUNCTION STATEMENTS
C
      DIMENSION XD(22),AIC(22,22),BIC(22,22),CIC(22,22)
      DIMENSION ARAI(22),ARM(22),ARAJ(22)
      DIMENSION ZIU(22,22),ZIL(22,22)
      DIMENSION AICP(22,22),BNON(22,1),DYN(22,22),DYNS(22,22),Y(22),
     1 YN(22),YN1S(22),SWPM(2,22),ARP(22)
      COMMON AICP,BNON
    5 CONTINUE
      READ 3,N
      IF(N) 160,160,6
    6 CONTINUE
      GG=32.174
      READ 3,IO
      IF(IO) 7,7,8
    8 CONTINUE
      READ 200,(ARAI(I),I=1,N)
      READ 200,(ARAJ(I),I=1,N)
      READ 200,(ARM(I),I=1,N)
      READ 200,(ARP(I),I=1,N)
      READ 200,(XD(I), I=1,N)
      READ 9,E,G,DEN,OM,R,R0
    9 FORMAT(3E10.4,3F10.2)
      WB=0.0
      DO 202 I=1,N
  202 WB=WB+ARM(I)
  200 FORMAT(5E12.5)
      PRINT 1030
 1030 FORMAT(1H1)
      PRINT 1022,DEN,E,G
      PRINT 1024,OM
      GO TO 12
    7 CONTINUE
```

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```
      READ 2,TCR,TCT,CR,CT
      READ 2,R,R0,ATP,ART
      READ 200, ZMIT,ZMIR,ZMJT,ZMJR,FJ
      READ 9,E,G,DEN,OM
      PRINT 1020,TCR,TCT,CR,CT,R0,R
      PRINT 1022,DEN,E,G
      PRINT 1024,OM
 1020 FORMAT(1H1,25X,$BLADE GEOMETRY$//5X,$ROOT T/C =$,F6.3,10X,$TIP T/C
     1 =$,F6.3/5X,$ROOT CHORD =$,F8.2,6X,$TIP CHORD =$,F8.2/5X,$ROOT RAD
     2IUS =$,F8.2,5X,$TIP RADIUS =$,F8.2,5X///)
 1022 FORMAT(25X,$MATERIAL PROPERTIES$//5X,$DENSITY =$,F12.6,7X,$E =$,1P
     3E13.5,7X,$G =$,1PE13.5///)
 1024 FORMAT(25X,$FLIGHT CONDITIONS$//5X,$OMEGA =$,F8.2)
    2 FORMAT(4F10.2)
    3 FORMAT(I5)
C   AVERAGE AREA INERTIAS AND MASS/UNIT LENGTH - INTEGRATION BY
C    FOUR POINT NEWTON-COTES FORMULA (SIMPSONS 3/8 RULE)
      AN = N
      RP = R-R0
      H = RP/(6.*AN)
      ARAI(1) = (AMI(R0)+3.*AMI(R0+H)+3.*AMI(R0+2.*H)+AMI(R0+3.*H))/8.
      ARAJ(1) = (AMJ(R0)+3.*AMJ(R0+H)+3.*AMJ(R0+2.*H)+AMJ(R0+3.*H))/8.
C   FJ IS AN EXPERIMENTALLY DETERMINED CORRECTION FACTOR TO OBTAIN J FROM THE
C     CALCULATED SECTION POLAR AREA MOMENT OF INERTIA
      ARAJ(1) = ARAJ(1)*FJ
      H = RP/(3.*AN)
      DO 10 I = 2,N
      J=2*I-3
      X0 = J*RP/(2.*AN) + R0
      ARAJ(I)=(AMJ(X0)+3.*AMJ(X0+H)+3.*AMJ(X0+2.*H)+AMJ(X0+3.*H))/8.
      ARAJ(I) = ARAJ(I)*FJ
   10 ARAI(I) = (AMI(X0)+3.*AMI(X0+H)+3.*AMI(X0+2.*H)+AMI(X0+3.*H))/8.0
      WB=0.0
      DO 20 I =1,N
      J=I-1
      X0 = J*RP/AN + R0
      ARM(I) = (AMS(X0)+3.*AMS(X0+H)+3.*AMS(X0+2.*H)+AMS(X0+3.*H))*RP/
     1 (8.*AN)
      ARP(I) = (AMJ(X0)+3.*AMJ(X0+H)+3.*AMJ(X0+2.*H)+AMJ(X0+3.*H))*DEN
     1  /(8.*GG)
   20 WB=WB+ARM(I)
      XD(1) = RP/(2.*AN) + R0
      H = RP/AN
      DO 25 I=2,N
   25 XD(I)=XD(I-1) + H
   12 CONTINUE
      PRINT 998
  998 FORMAT(1H1,10X,$BLADE CHARACTERISTICS AT COLLOCATION POINTS$/)
      PRINT 1000
 1000 FORMAT(// 10X,$X$,14X,$IXX(X)$,13X,$WT(X)  $,18X,$J(X)$,11X,$IZZ(X
     1),POLAR$)
      DO 30 I =1,N
   30 PRINT 1001,XD(I),ARAI(I),ARM(I),ARAJ(I),ARP(I)
 1001 FORMAT(6X,F6.3,4E20.4)
      PRINT 999,WB
  999 FORMAT(//10X,$BLADE WEIGHT =$,F10.2)
C   OBTAIN INFLUENCE COEFFICIENTS FOR DISPLACEMENT,SLOPE, AND TORSIONAL
C     DEFLECTION
C     K DENOTES LOADED STATION
      AN = N
      DO 80 K=1,N
      I=1
      C1=0.0
      C2=0.0
      GO TO 65
   60 C1=BIG(I,K)*F*(ARAI(I+1)-ARAI(I))+C1
```

```
      C2=(AIC(I,K)-BIC(I,K)*XD(I))*E*(ARAI(I+1)-ARAI(I))+C2
      I=I+1
   65 BIC(I,K)=(XD(K)*XD(I)-XD(I)**2/2.0+C1)/(E*ARAI(I))
      AIC(I,K)=(XD(K)*XD(I)*XD(I)/2.0-XD(I)**3/6.0+XD(I)*C1+C2)/(E*ARAI(
     1I))
      IF(I-K) 60,70,70
   70 L=I+1
      DO 75 J=L,N
      BIC(J,K)=BIC(K,K)
      AIC(J,K)=AIC(K,K)+BIC(K,K)*(XD(J)-XD(K))
   75 CONTINUE
   80 CONTINUE
C   TORSIONAL FLEXIBILITY MATRIX, INFLUENCE COEFFICIENTS
      DO 400 J=1,N
      DO 400 I=1,N
      IF(I-J) 402,402,404
  402 ZIU(I,J) = 1.0
      GO TO 406
  404 ZIU(I,J) = 0.0
  406 ZIL(J,I) = ZIU(I,J)
  400 CONTINUE
C   DELX = LENGTH OF FLEXIBLE CONNECTION BETWEEN MASSES
C   DXX = LENGTH OF ELEMENT WITH MASS PROPERTIES
      DELX = XD(1) - R0
      DXX = 2.*(XD(1)-R0)
      DO 420 J = 1,N
      DO 422 I = 1,N
  422 DYN(I,J) = 0.0
      DYN(J,J) = DELX/(ARAJ(J)*G)
      ARP(J) = ARP(J)*DXX
      DXX = 2.*(XD(J+1)-XD(J)) - DXX
  420 DELX = XD(J+1)-XD(J)
      CALL ZATRIX(DYN,ZIU,DYNS,N,N,N)
      CALL ZATRIX(ZIL,DYNS,CIC,N,N,N)
C   PRINT ROUTINE
   83 PRINT 1005
 1005 FORMAT(1H1,30X,$I N F L U E N C E  C O E F F I C I E N T  M A T R
     1I C E S $///,40X,$DEFLECTION - AIC(I,J)$/)
      LK=1
      KP=N/10
   85 KON=0
      NN1=1
      N1=1
      N2=10
   86 IF(KP-NN1) 87,88,89
   87 N2=N
   84 KON=2
      GO TO 89
   88 IF(MOD(N,10)) 84,84,89
   89 DO 93 I=1,N
      GO TO (90,91,92),LK
   90 PRINT 1007,(AIC(I,J), J=N1,N2)
      GO TO 93
   91 PRINT 1007,(BIC(I,J),J=N1,N2)
      GO TO 93
   92 PRINT 1007,(CIC(I,J), J=N1,N2)
   93 CONTINUE
 1007 FORMAT(5X,10E12.5)
      NN1=NN1+1
      N1=N1+10
      N2=N2+10
      IF(KON-1) 94,94,95
   94 PRINT 1009
 1009 FORMAT(/30X,$- - - - - - CONTINUING ACROSS, INCREASING J - - - - -
     1 -$/)
      GO TO 86
```

```
   95 LK = LK + 1
      GO TO (97,97,99,101),LK
   97 PRINT 1011
 1011 FORMAT(1H1,40X,$SLOPE - BIC(I,J)$/)
      GO TO 85
   99 PRINT 1013
 1013 FORMAT(1H1,40X,$TORSION - CIC(I,J)$/)
      GO TO 85
  101 CONTINUE
C
C   CALCULATION OF D MATRIX
C
      DO 207 I=1,N
  207 ARM(I)=ARM(I)/GG
C     (AM) MATRIX
  230 DO 235 J=1,N
      DO 237 I=1,N
  237 AIC(I,J)=ARM(J)*AIC(I,J)
  235 CONTINUE
      IF(OM) 240,240,205
C     (A C) MATRIX
  205 CONST = OM**2
      DO 210 K=1,N
      DO 220 I=1,N
  220 AICP(I,K)=CONST*AIC(I,K)
  210 CONTINUE
C     (I + AC) MATRIX
      DO 225 K=1,N
      AICP(K,K)=1.0+AICP(K,K)
  225 BNON(K,1)=1.0
      CALL MATINV(N,1,DETERM,ID)
      IF(ID-1) 245,245,227
  227 PRINT 2111
 2111 FORMAT(2X,$****NO MATRIX INVERSE ****$)
      GO TO 360
C     DYN MATRIX
  240 DO 242 I=1,N
      DO 242 J=1,N
  242 DYN(I,J) = AIC(I,J)
      GO TO 253
  245 DO 247 I=1,N
      DO 247 J=1,N
      DYN(I,J) = 0.0
      DO 250 K=1,N
  250 DYN(I,J)=DYN(I,J)+AICP(I,K)*AIC(K,J)
  247 CONTINUE
C
C  MATRIX ITERATION FOR OMEGA AND MODE SHAPE
C
C     INITIAL MODE SHAPE
  253 MS=1
  255 DO 260 I=1,N
  260 Y(I)=DYN(I,N)/DYN(N,N)
C     ITERATION
      SLAM=0.0
      KOK=0
  262 DO 265 I=1,N
      YN(I)=0.0
      DO 265 J=1,N
  265 YN(I)=DYN(I,J)*Y(J)+YN(I)
      ALAMDA = YN(N)
C     NORMALIZE MODE SHA E
      DO 270 I=1,N
  270 YN(I)=YN(I)/YN(N)
C     CONVERGENCE CHECK
      IF(ABS("'AMDA-SLAM)-.0001*ALAMDA) 305,275,275
```

```
  275 SLAM = ALAMDA
      DO 280 I=1,N
  280 Y(I)=YN(I)
      KQK=KQK+1
      IF(KQK-20) 262,262,285
  285 PRINT 1034, KQK
 1034 FORMAT(1H1//////20X,$*** NO CONVERGENCE AT$I3$ ITERATIONS***$)
  305 CONTINUE
  300 FRQ = 1.0/SQRT(ALAMDA)
      PRINT 1035,MS,FRQ ,KQK
 1035 FORMAT(1H1,20X,$MODE SHAPE$,I3,5X,$NAT. FREQ =$,E12.5,5X$ITERATION
     1S =$,I3/)
      PRINT 1040 ,(YN(I),I=1,N)
 1040 FORMAT(10(/5X,F12.8))
      GO TO (320,350,360,360),MS
C
C  SECOND MODE SHAPE
C
  320 MS=2
C     SWEEPING MATRIX
      SWPM(1,1)=0.0
      DO 330 J=2,N
  330 SWPM(1,J)=-YN(J)*ARM(J)/(ARM(1)*YN(1))
C     SAVE FIRST MODE SHAPE AND DYN MATRIX
      DO 335 I=1,N
      YN1S(I)=YN(I)
      DO 335 J=1,N
  335 DYNS(I,J)=DYN(I,J)
C     NEW DYN MATRIX FOR SECOND MODE
  340 L=MS-1
      DO 345 I=1,N
      DYN(I,L) =0.0
      DO 345 J=MS,N
      DYN(I,J)=DYNS(I,J)
      K=1
  342 DYN(I,J) = DYN(I,J)+DYNS(I,K)*SWPM(K,J)
      K=K+1
      IF(K-L) 342,342,345
  345 CONTINUE
      GO TO 255
C
C  THIRD MODE SHAPE
C
  350 MS=3
       SWPM(1,2)=0.0
      SWPM(2,1)=0.0
      SWPM(2,2)=0.0
      DNOM1=ARM(1)*(YN(2)*YN1S(1)-YN1S(2)*YN(1))
      DNOM2=ARM(2)*(YN1S(2)*YN(1)-YN1S(1)*YN(2))
      DO 355 J=3,N
      SWPM(1,J)=(YN(J)*YN1S(2)-YN1S(J)*YN(2))*ARM(J)/DNOM1
  355 SWPM(2,J)=(YN(J)*YN1S(1)-YN1S(J)*YN(1))*ARM(J)/DNOM2
C     NEW DYN MATRIX FOR THIRD MODE
      GO TO 340
C
C  TORSIONAL FREQUENCIES
C
C  FIRST MODE
  360 MS = 4
      DO 450 I = 1,N
      DO 440 J = 1,N
  440 DYNS(I,J) = 0.0
  450 DYNS(I,I) = ARP(I)
      CALL ZATRIX(CIC,DYNS,DYN,N,N,N)
C  INITIAL MODE SHAPE
  500 DO 501 I=1,N
```

```
501 Y(I) = DYN(I,N)/DYN(N,N)
    KQK = 0
C   ITERATION
502 DO 504 I = 1,N
    YN(I) = 0.0
    DO 504 J = 1,N
504 YN(I) = DYN(I,J)*Y(J) + YN(I)
    KONV = 1
C   NORMALIZE AND CHECK CONVERGENCE
    DO 508 I = 1,N
    YN(I) = YN(I)/YN(N)
    IF(ABS(YN(I)-Y(I))-.0001*YN(I)) 508,508,506
506 KONV = 0
508 CONTINUE
    IF(KONV) 510,510,370
510 DO 512 I = 1,N
512 Y(I) = YN(I)
    KQK = KQK + 1
    IF(KQK-30) 502,502,514
514 PRINT 1034, KQK
370 DENOM = 0.0
    MSS = MS-3
    DO 460 J = 1,N
460 DENOM = DYN(N,J)*YN(J) + DENOM
    FRQ = SQRT(YN(N)/DENOM)
    PRINT 1050,MSS,FRQ,KQK
1050 FORMAT(1H1,10X,$TORSION MODE$,I3,5X,$NAT. FREQ =$,E12.5,5X,$ITERAT
    1IONS =$,I3/)
    PRINT 1040,(YN(I), I = 1,N)
    GO TO 5
160 CONTINUE
    STOP
    END
```

```
      SUBROUTINE MATINV(N1,M1,DETERM,ID)
C
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C     NOVEMBER 1692   S GOOD  DAVID TAYLOR MODEL BASIN   AM MAT1
C
C     GENERAL FORM OF DIMENSION STATEMENT
C     DIMENSION    A(  ,  ),B(  ,  ),INDEX(  ,3)
C
      DIMENSION A(22,22),B(22,1), INDEX(22,3)
      COMMON A,B
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C
C     INITIALIZATION
C
      M=M1
      N=N1
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 INDEX(J,3) = 0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
      IF(INDEX(J,3)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF(INDEX(K,3)-1) 80, 100, 715
   80 IF (      AMAX -ABS (A(J,K))) 85, 100, 100
   85 IROW=J
   90 ICOLUM=K
      AMAX = ABS (A(J,K))
  100 CONTINUE
  105 CONTINUE
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) +1
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140, 310, 140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF(M) 310, 310, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  310 PIVOT    =A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVCT
  330 A(ICOLUM,ICOLUM)=1.0
```

```
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      DO 730 K = 1,N
      IF(INDEX(K,3) -1) 715,720,715
  715 ID =2
      GO TO 740
  720 CONTINUE
  730 CONTINUE
      ID =1
  740 RETURN
C     LAST CARD OF PROGRAM
      END
```

```
      SUBROUTINE ZATRIX(A,B,C,N,L,M)
      DIMENSION A(22,22),B(22,22),C(22,22)
      DO 30 I=1,N
      DO 20 J=1,M
      C(I,J)=0.0
      DO 10 K=1,L
      C(I,J) = C(I,J) + A(I,K)*B(K,J)
   10 CONTINUE
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
```

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